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mathematics will have a good effect in creating a greater demand for such literature. Two recent American publications which may reasonably be expected to exert a good influence along this line are J. W. Young's *Fundamental Concepts of Algebra and Geometry,* and J. W. A. Young's *Monographs on Modern Mathematics.* These were published by the Macmillan Company, and by Longmans, Green and Company, respectively.

NOTE ON THE BINOMIAL SERIES.

By K. OGURA in Sendai, Japan.

Euler proved the binomial theorem

$$F(y)=(1+x)^y=\sum_{n=0}^{\infty}\frac{y(y-1)\ldots\ldots(y-n+1)}{n!}x^n \qquad |x|<1,$$

provided that y is a rational number. There are various proofs of the above formula when y is irrational.

But it is hoped that the following proof will be found both simple and rigorous.

In the binomial series

$$F(y)=\sum_{n=0}^{\infty}\frac{y(y-1)\ldots\ldots(y-n+1)}{n.}x^n \qquad |x|<1,$$

put

$$f_n(y)=\frac{y(y-1)\ldots\ldots(y-n+1)}{1.2\ldots n}.$$

Let us denote by G a finite positive number however great, and let us take y so that $-G<y<G$.

(i) If $y>-1$, we can choose the positive numbers k and n so that $y+1<k<n$. Then

$$0<\frac{y+1}{k}<1, \quad 0<\frac{y+1}{k+1}<1, \ldots, 0<\frac{y+1}{n}<1, \ldots$$

and

$$|f_n(y)|=\left|\left(1-\frac{y+1}{1}\right)\left(1-\frac{y+1}{2}\right)\ldots\left(1-\frac{y+1}{k-1}\right)\right|\times\left(1-\frac{y+1}{k}\right)\ldots\left(1-\frac{y+1}{n}\right).$$

But since

$$\lim_{n=\infty} \sum_{\nu=1}^{n} \frac{y+1}{\nu} = +\infty,$$

we have

$$\prod_{n=k}^{\infty} \left(1 - \frac{y+1}{n}\right) = 0.$$

Hence

$$\lim_{n=\infty} | f_n(y) | = 0.$$

Therefore we can choose n so that $| f_n(y) | < A$, A being a fixed positive number.

(ii) If $y=-1$, then

$$| f_n(y) | = 1.$$

(iii) If $y<-1$, we can choose a positive number p, independent of y, so that $-y<1+p<G$. Then

$$| f_n(y) | < (1+\frac{p}{1})\,(1+\frac{p}{2})\dots(1+\frac{p}{n}) < e^p.e^{\frac{1}{2}p}\dots e^{p/n}$$

$$= e^{p(1+\frac{1}{2}+\dots+1/n)} < e^{p(C+\log n)} = e^{pC}.n^p,$$

C being Euler-Mascheroni's constant.

If we put $A=e^{pC}$, then $| f_n(y) | < A.n^p$.

Hence, in general, there exists a positive number $X=1$ such that

$$| f_n(y) | X^n < An^p$$

for fixed positive A and p.

Therefore, by a theorem due to Pringsheim,* it follows that the series $F(y)$ is a continuous function of y. $(-G<y<G)$.

Now if an irrational number y be represented as the limit of the sequence $(y_1, y_2, \dots, y_n, \dots)$ whose terms are all rational, then we have

$$F(y) = \lim_{n=\infty} F(y_n) = \lim_{n=\infty} (1+x)^{y_n}.$$

But by the definition of the power

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* Pringsheim, *Ber. München*, 27 (1897), p. 35; Bromwich, *Infinite Series*, 1908, p. 135.

$$\lim_{n=\infty}(1+x)^{y_n}=(1+x)^y.$$

Hence we have $F(y)=(1+x)^y$; that is,

$$(1+x)^y=1+\frac{y}{1\,!}x+\frac{y(y-1)}{2!}x^2+\ldots \qquad |\,x\,|<1.$$

Thus the binomial theorem is proved for all finite values of y.

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MOMENT OF INERTIA OF A RING CALCULATED BY AN ELEMENTARY METHOD.

By B. H. BROWN, Whitman College, Walla Walla, Washington.

The problem of finding the moment of inertia of a homogeneous circular ring, of unit density and of radius R with cross-section πr^2, when referred to a diameter, appears to be quite generally avoided by our writers on Elementary Mechanics. The following solution may prove of interest.

Suppose the ring to be edgewise to the plane of the paper and bisected by the plane of the paper giving the two circles of the figure as cross-sections of the ring. Plane PQ perpendicular to the paper bisects the ring and contains the axis of rotation, PQ passing through O, the center of the ring. The moment of inertia of the central section containing the plane PQ and of thickness dy is given by

$$I_c=\frac{\pi}{4}(A^4-a^4)dy. \quad \text{But } A=R+(r^2-y^2)^{\frac{1}{2}} \text{ and } a=R-(r^2-y^2)^{\frac{1}{2}}.$$

$$\therefore I_c=\frac{\pi}{4}\{[R+(r^2-y^2)^{\frac{1}{2}}]^4-[R-(r^2-y^2)^{\frac{1}{2}}]^4\}dy$$

$$=2\pi R^3(r^2-y^2)^{\frac{1}{2}}\,dy+2\pi R(r^2-y^2)^{\frac{3}{2}}\,dy.$$

The moment of inertia for any other section as BF parallel to PQ at a distance y from the axis is of course given by $I_y=I_c+my^2$.

The area $BDEF$ cut from the ring by plane BF is equal to

$$\pi\,[R+(r^2-y^2)^{\frac{1}{2}}]^2-\pi\,[R-(r^2-y^2)^{\frac{1}{2}}]^2 \text{ or } 4\pi R(r^2-y^2)^{\frac{1}{2}},$$